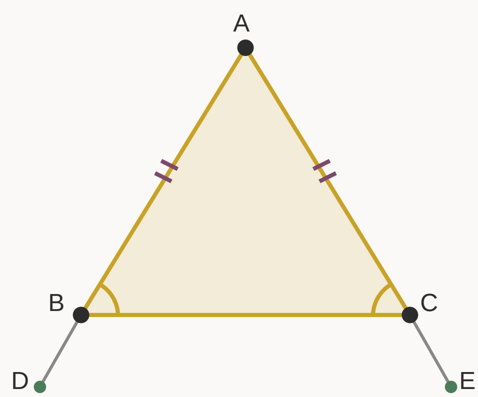
Worksheet: Proposition 5

In isosceles triangles the angles at the base equal one another; and if the equal sides are produced further, the angles under the base will equal one another.



Isosceles triangle ABC (AB = AC): base angles at B and C are equal

Student Name: ________________________ Date: ____________

Part A: Construction Practice

Draw and label the diagram described in Proposition 5. Mark all given information.

After completing, answer:

1. What tools did you use, and in what order?

2. Which postulates justify each step of your construction?

Part B: Proof Justification

Below is the proof broken into steps. For each step, write the toolkit item that justifies it.

Available Toolkit:

• D20 — Types of Triangles (by sides)

• P2 — Extend a Line

• Prop 3 — To cut off from the greater of two given unequal straight lines a straight line equal to the less.

• Prop 4 — If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively.

• CN1 — Transitivity of Equality

• CN2 — Addition Preserves Equality

• CN3 — Subtraction Preserves Equality

• P1 — Draw a Line

Step	Statement	Justification
1	Given: Triangle ABC with AB = AC (isosceles)	
2	Extend AB to D and AC to E.	
3	Make BD = CE.	
4	Since AB = AC and BD = CE, then AB + BD = AC + CE.	
5	Therefore AD = AE.	
6	Join DC and EB.	
7	In triangles ADC and AEB: AD = AE, AC = AB, and DAC = EAB (the angle between the same two lines through A). Therefore DC = EB, ACD = ABE, and ADC = AEB.	
8	In triangles BDC and CEB: BD = CE, DC = EB, and BDC = CEB (equal to ADC = AEB, since B lies on AD and C lies on AE). Therefore DBC = ECB and BCD = CBE.	
9	Subtracting the equal angles CBE = BCD from the equal whole angles ABE = ACD gives ABC = ACB.	

Part C: Thinking Deeper

Question 1: Why is this called the 'Bridge of Asses'? What makes it a turning point?

Question 2: Can you think of a visual or physical way to see that the base angles must be equal?

Question 3: What would it mean if an isosceles triangle had UNEQUAL base angles?

Part D: Challenge Problem

Can you think of a situation where Proposition 5 would be useful in proving something else? Describe it below.

Part E: Self-Assessment

Reflect honestly on your understanding. There are no wrong answers here.

1. Could you explain to someone else why this proof works, without looking at your notes?

- ☐ Yes, I could explain the whole proof
- ☐ I could explain most of it but might get stuck
- ☐ I'd need to look at the steps again

2. Which part of this lesson was hardest for you?

☐ Drawing the construction / diagram

☐ Understanding why the proof works (the logic)

☐ Matching proof steps to toolkit items

☐ The extension questions

3. What would you do differently next time?
