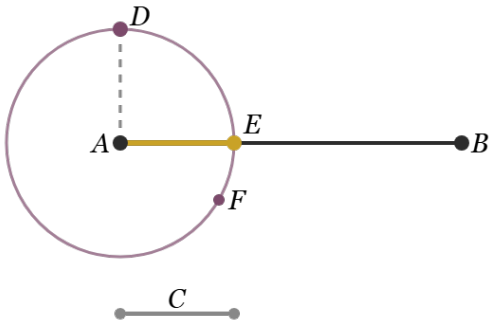
Lesson Plan: Proposition 3

To cut off from the greater of two given unequal straight lines a straight line equal to the less.

Grade: 9-12 | Duration: 45-55 minutes | Type: Construction



You have a long line and a short line. You need to mark a spot on the long line so the piece from one end to the mark is exactly the length of the short line. This completes our basic toolkit: Prop 1 builds equal sides, Prop 2 copies lengths anywhere, and Prop 3 trims to size. Notice how quick this construction is compared to Prop 2 — that's the payoff for doing the hard work earlier. Once you have the right tools, new results come fast.

Stage 1: Desired Results (UbD)

Essential Question

How do simple tools, combined systematically, give us precise control over geometric quantities?

Learning Objectives

1. Given two unequal segments, cut the longer so that a piece equals the shorter
2. Apply Proposition 2 as a sub-procedure to copy the shorter length to an endpoint of the longer segment
3. Explain how Propositions 1-3 together form a complete toolkit for working with lengths
4. Justify the construction using the definition of a circle (D15) and Common Notion 1

Key Understandings

• Propositions 1, 2, and 3 form a chain: each one depends on the previous and expands what we can do with lengths

• A circle centered at a point acts as a 'length stamp' — any radius of that circle has the same length

• In Euclidean geometry, we build complex capabilities from simple postulates one step at a time

Vocabulary

Cut off (in the Euclidean sense), Arc, Radius, Sub-construction, Given (as a term in proofs)

Stage 2: Assessment Evidence

Formative Checks

• Before starting: can your student explain in one sentence what Prop 2 lets us do? If not, review before moving on

• After the construction: use a ruler to verify the cut segment matches the short one — then discuss why measurement confirms but doesn't prove

• Quick-write: 'Explain this construction to someone who knows Props 1 and 2 but hasn't seen Prop 3'

Materials Needed

• Compass and straightedge for each student

• Blank unlined paper

• Printed worksheet with pairs of unequal segments

• Rulers (for verification only, not for construction)

Stage 3: Learning Plan

Warm-Up (5 minutes)

Draw two segments on the board: a long one (AB) and a short one (CD). Ask: 'How would you mark a point on AB so the piece from A to that point is exactly the length of CD? No measuring.' Give a minute to think. Most students will suggest using Prop 2 to copy CD to point A — and that's exactly right. Confirm the instinct.

Direct Instruction (10 minutes)

Here's how it works: (1) We have long segment AB and short segment CD. (2) Use Prop 2 to copy the length of CD to point A — call the new segment AE, so AE = CD. (3) Draw a circle centered at A with radius AE. (4) This circle crosses AB at a point F. (5) AF = AE because they're both radii of the same circle (Definition 15). (6) AE = CD by construction (Prop 2). (7) So AF = CD (Common Notion 1). Done — we've trimmed AB to match CD. Notice how short this is compared to Prop 2. That's the reward for building good tools first.

Guided Practice (12 minutes)

Work through two examples on the worksheet. For the first, use the step-by-step scaffold with sentence starters for each justification. For the second, write the construction and justification from scratch. If working with a partner, check each other's work. Key questions to think about: which step uses Prop 2? Which uses D15?

Independent Practice (15 minutes)

Worksheet: (a) Do the Prop 3 construction with a new pair of segments. (b) Given three segments of different lengths, use Prop 3 twice to cut the longest down to the shortest. (c) In one sentence each, summarize what Prop 1, Prop 2, and Prop 3 let you do. (d) Predict: what kinds of problems can we solve now that we have all three tools?

Closure (8 minutes)

Let's build a toolkit chart: Prop 1 = build equal sides. Prop 2 = copy a length anywhere. Prop 3 = trim to a desired length. With these three, we have complete control over lengths. But what about angles? We don't have any angle tools yet. That's about to change — Proposition 4 is the first theorem (not a construction), and it proves something powerful about when two triangles must be identical.

Differentiation: Supporting All Learners

For Struggling Students

Since Prop 3 is straightforward once Prop 2 makes sense, focus support on the prerequisite. Provide a completed Prop 2 construction as the starting point so your student can concentrate on the new circle-intersection step. Color-code: red for the short segment, blue for the long one, green for the copied length.

For Advanced Students

What if the two segments are equal? What does the construction produce? Can you think of a real-world situation where you need to cut something to match a specific length without a measuring tool? Why did Euclid order Props 1, 2, 3 in this sequence?

Tips: Teacher Notes

> **Teacher note:** This lesson feels almost too easy after Prop 2. That's a feature, not a bug — use it to show that good foundations make later work simpler.

> **Teacher note:** Watch for students who skip the Prop 2 step and try to 'just use the compass to copy CD directly.' Remind them about the collapsing compass.

> **Teacher note:** If students are shaky on Props 1-2, this is a natural checkpoint. Spend the independent practice on review rather than pushing forward.

> **Teacher note:** Keep the toolkit chart posted — it becomes a reference students reach for throughout the unit.

Preview: Next Lesson

Next: Proposition 4 — If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, then they also have the base equal to the base, the triangle equals the triangle, and the remaining angles equal the remaining angles respectively.

If two triangles have two sides the same length and the angle between those sides is also equal (SAS - Side-Angle-Side), then the triangles are identical in every way.