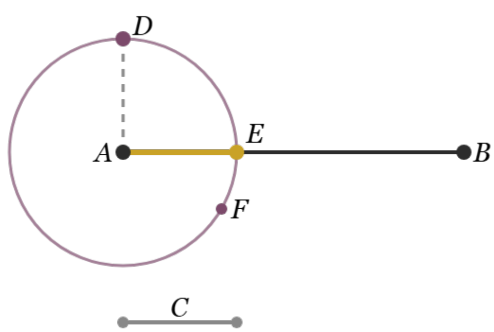
Answer Key: Proposition 3

To cut off from the greater of two given unequal straight lines a straight line equal to the less.

Part A: Construction Practice — Notes

Solution Figure



Teacher note: This construction is shorter than Prop 2 but depends on it. Given two unequal lines AB (longer) and C (shorter): (1) use Prop 2 to place a segment AD equal to C, starting at point A; (2) draw a circle centered at A with radius AD (P3); (3) mark the intersection point E where this circle crosses AB; (4) note AE = AD because both are radii of the same circle (D15); (5) therefore AE = C (CN1). Have students perform the Prop 2 construction explicitly at least once, so they understand the dependency rather than treating it as a black box.

Part B: Proof Justification — Answers

Each step is matched with the correct toolkit justification.

Available Toolkit:

• P3 — Draw a Circle

• D15 — Circle

• CN1 — Transitivity of Equality

• Prop 2 — To place a straight line equal to a given straight line with one end at a given point.

• P1 — Draw a Line

• Prop 1 — To construct an equilateral triangle on a given finite straight line.

• CN3 — Subtraction Preserves Equality

Step	Statement	Justification
1	Place a line AD at point A equal to the shorter line C	Prop 2
2	Draw a circle centered at A with radius AD; let E be where it meets AB	P3
3	AE = AD (both radii of the circle centered at A)	D15
4	Since AD = C (by construction), we have AE = C	CN1

Teacher note: The proof works by combining Prop 2 (length transfer) with a circle (equal radii). Prop 2 copies length C to point A, creating segment AD where AD = C. The circle centered at A with radius AD then marks off exactly that distance along line AB. Point E, where the circle intersects AB, satisfies AE = AD (both are radii of the circle centered at A, by D15). Since AD = C (from Prop 2) and AE = AD (radii), CN1 gives us AE = C. The greater line AB has been trimmed to match the lesser line C.

Why each justification works:

Step 1: Prop 2 — To place a straight line equal to a given straight line with one end at a given point.. Which proposition lets us copy a length to a new location?

Step 2: P3 — Draw a Circle. Which postulate lets us draw a circle?

Step 3: D15 — Circle. What definition tells us all radii of a circle are equal?

Step 4: CN1 — Transitivity of Equality. Things equal to the same thing are equal to each other.

Part C: Thinking Deeper — Model Answers

Question 1: Why do we need Proposition 2 first before we can do this?

> **Teacher note:** Prop 2 is essential because our compass can only draw circles through existing points — it cannot be "preset" to an arbitrary length. If segment C is not located at point A, we have no way to draw a circle of radius C centered at A without first transferring that length. Prop 2 provides exactly this transfer capability.

Question 2: What if the shorter line were longer than AB? Would this construction fail?

> **Teacher note:** No. Doing it "by eye" provides no logical guarantee that the lengths are truly equal. Euclidean geometry demands that every equality be justified by postulates, definitions, or common notions. The circle is what guarantees exactness — every point on it is provably equidistant from the center. Visual estimation, no matter how careful, is not a proof.

Question 3: How is this different from just using a ruler to measure?

> **Teacher note:** Props 1 through 3 form a chain of increasing capability. Prop 1 constructs equal lengths (the equilateral triangle). Prop 2 uses Prop 1 to copy any length to any location. Prop 3 uses Prop 2 to cut a segment to match a given length. Together, they give Euclid complete control over line segments — he can create, relocate, and trim them at will. Every later construction in Book I relies on this trio.

Part D: Challenge Problem — Model Answer

> **Teacher note:** To cut a segment equal to C from the middle of AB (not starting at endpoint A), you would first pick a point M on AB, then apply Prop 2 to copy length C to point M, and finally apply Prop 3 using M as the endpoint. This requires combining Props 2 and 3 in sequence and demonstrates that the tools Euclid has built are composable — they work together to handle situations beyond what any single proposition addresses.

Notes: Common Student Mistakes

• Skipping the Prop 2 step and jumping straight to drawing a circle with radius C. Students must recognize that we cannot simply "set" the compass to length C — we need Prop 2 to physically relocate that length to point A first.

• Confusing the two uses of D15 in steps 3 and 4. Both steps invoke the definition of a circle (all radii are equal), but step 3 identifies point E as lying on the circle, while step 4 states the equality AE = AD. They are logically distinct moments in the proof.

• Writing P1 for step 1 instead of citing Prop 2. The first step is not drawing a line — it is performing the entire Prop 2 construction to copy length C to point A.

• Not understanding why the proposition requires AB to be longer than C. If C were longer than AB, the circle centered at A with radius AD would not intersect segment AB between A and B.