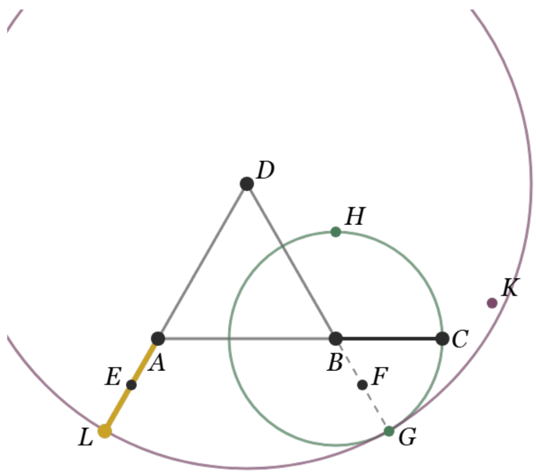
Lesson Plan: Proposition 2

To place a straight line equal to a given straight line with one end at a given point.

Grade: 9-12	Duration: 45-55 minutes	Type: Construction



Here's the puzzle: you have a line segment BC and a point A somewhere else on the page. You need to create a new segment starting at A that's exactly the same length as BC. The catch? Your compass 'collapses' when you lift it — you can't just measure BC and redraw it at A. This lesson shows how the equilateral triangle from Proposition 1 acts as a relay station to transfer lengths anywhere. It's the most construction-heavy of the first five propositions, but once you see the trick, it's elegant.

Stage 1: Desired Results (UbD)

Essential Question

When our tools have limitations, how can we combine simple constructions to accomplish something none of them can do alone?

Learning Objectives

1. Construct a segment equal in length to a given segment, starting from a specified point
2. Explain why a collapsing compass requires an indirect method to transfer lengths
3. Use Proposition 1 as a sub-procedure within a longer construction
4. Apply Common Notions 1 and 3 to justify each step of the proof

Key Understandings

• Euclid's compass collapses when lifted — you cannot directly carry a length from one place to another

• An equilateral triangle acts as a bridge between two points, letting us extend and then cut a length along a known direction

• Complex constructions are built by chaining simpler ones, just like complex theorems build on simpler results

Vocabulary

Collapsing compass, Equilateral triangle, Common Notion 3 (subtraction), Postulate 2 (extend a line), Postulate 3 (draw a circle), Sub-construction

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Stage 2: Assessment Evidence

Formative Checks

• Before starting, ask: 'Why can't we just open the compass to length BC and move it to point A?' Make sure the collapsing-compass constraint is clear

• Mid-construction: have your student label the equilateral triangle and confirm all three sides are equal before moving on

• After finishing, have one student explain steps 1-3 and another explain steps 4-6, then both verify the result

Materials Needed

• Compass and straightedge for each student

• Blank unlined paper
• Printed worksheet with a given segment and a separate target point
• Highlighters in two colors (for distinguishing the equilateral triangle from the final construction)

Stage 3: Learning Plan

Warm-Up (5 minutes)

Draw a segment BC on the left side of the paper and mark a point A on the right side. Here's the challenge: 'Your compass collapses the moment you lift it. You can't measure with a ruler. How would you create a segment starting at A that's exactly the same length as BC?' Take a couple of minutes to think about it. This is genuinely hard — the difficulty is the whole point. It motivates the clever relay method Euclid invented.

Direct Instruction (15 minutes)

Walk through the construction step by step — use two colors, one for the relay triangle and one for the circles. (1) Connect A to B with a straight line. (2) Build an equilateral triangle DAB on that line using Prop 1. Now DA = DB = AB. (3) Extend the line DA beyond A, and extend DB beyond B. (4) Draw a circle centered at B passing through C — where it hits the extension of DB, call that point G. Now BG = BC. (5) Draw a circle centered at D passing through G — where it hits the extension of DA, call that L. Now DL = DG. (6) Here's the punchline: DG = DB + BG, and DL = DA + AL. Since DA = DB (equilateral triangle) and DL = DG (both radii), subtract: AL = BG = BC. Done.

Guided Practice (12 minutes)

Your student's turn. Using the worksheet with a pre-printed point and segment, work through the construction. If working with a partner, one person constructs while the other reads the steps aloud and checks each one. Then switch roles with a new point and segment. The equilateral triangle should connect A to B — watch for the common mistake of extending lines in the wrong direction.

Independent Practice (12 minutes)

Worksheet: (a) Copy a given segment to a given point from memory — no step-by-step guide. (b) In your own words, why do we need the equilateral triangle? What job does it do? (c) A completed Prop 2 figure is shown with missing labels — fill them all in and state which segments are equal and why. (d) How does this proposition depend on Proposition 1? Could we do it without Prop 1?

Closure (6 minutes)

Think about what we've unlocked: we can now place any length at any point. That's a huge capability from just two propositions. Looking ahead: 'What if we need to cut a longer line down to match a shorter one?' That's Proposition 3, and now that we have Prop 2, it's going to be much simpler.

Differentiation: Supporting All Learners

For Struggling Students

Provide a step-by-step visual guide with numbered diagrams showing each stage of the construction. Have your student trace over a lightly printed solution first, then try it on blank paper. Use different colored pencils for the triangle vs. the circles to keep things clear.

For Advanced Students

Some shorter constructions exist that also work — can you find one? Also: Prop 2 uses Prop 1 as a sub-construction. If you count every circle and line including the Prop 1 steps, how many total operations does the entire construction require? Think about efficiency.

Tips: Teacher Notes

> **Teacher note:** This is the hardest proposition so far. The relay-station idea is not obvious. Spend extra time on why we can't just carry the compass — that constraint makes the whole construction necessary.

> **Teacher note:** Draw the equilateral triangle in a distinct color so it stands out as scaffolding, not the final product.

> **Teacher note:** Watch for confusion about which lines to extend. It's through D, beyond A (not beyond B). Arrows showing direction help.

> **Teacher note:** Good analogy: Prop 2 is like a function that calls Prop 1 as a subroutine. Each proposition builds a library of tools you can reuse.

Preview: Next Lesson

Next: Proposition 3 — To cut off from the greater of two given unequal straight lines a straight line equal to the less.

Given a long line and a short line, mark a segment on the long line that equals the short line's length.