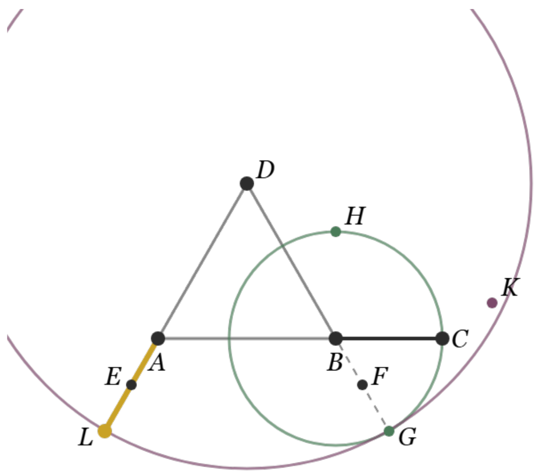
Answer Key: Proposition 2

To place a straight line equal to a given straight line with one end at a given point.

Part A: Construction Practice — Notes

Solution Figure



Teacher note: This is the most construction-heavy of the first five propositions. Walk students through it in stages: (1) connect A to B with a straight line (P1), (2) construct equilateral triangle DAB using Prop 1, (3) extend lines DA and DB beyond A and B respectively (P2), (4) draw a circle centered at B with radius BC (P3) to find point G on the extension of DB, (5) draw a circle centered at D with radius DG (P3) to find point L on the extension of DA. The result is AL = BC. Encourage students to draw each step in a different color to keep track of the accumulating construction.

Part B: Proof Justification — Answers

Each step is matched with the correct toolkit justification.

Available Toolkit:

- P1 — Draw a Line
- P2 — Extend a Line
- P3 — Draw a Circle
- Prop 1 — To construct an equilateral triangle on a given finite straight line.
- CN1 — Transitivity of Equality
- CN3 — Subtraction Preserves Equality
- D15 — Circle
- D20 — Types of Triangles (by sides)

Step	Statement	Justification
1	Connect A to B with a straight line	P1
2	Construct equilateral triangle DAB on line AB, giving DA = DB = AB	Prop 1
3	Extend DA beyond A, and extend DB beyond B	P2
4	Draw circle centered at B with radius BC — mark G where it crosses DB extended.	P3
5	Because G lies on the circle centered at B, BG = BC.	D15
6	Draw circle centered at D with radius DG — mark L where it crosses DA extended.	P3
7	Because L lies on the circle centered at D, DL = DG.	D15
8	DL = DG, with DL = DA + AL and DG = DB + BG. Since DA = DB, the remainders are equal: AL = BG.	CN3
9	AL = BG and BG = BC, therefore AL = BC.	CN1

> **Teacher note:** The proof transfers length BC to point A using the equilateral triangle as a relay. Since DAB is equilateral, DA = DB. The circle at B with radius BC gives BG = BC (D15). The circle at D with radius DG gives DL = DG (D15). Now DL = DG, and DA = DB (equilateral triangle), so DL - DA = DG - DB, which means AL = BG (CN3, subtracting equals from equals). Since BG = BC, we get AL = BC (CN1). The length has been successfully copied from B to A without ever "carrying" a distance with the compass.

Why each justification works:

Step 1: P1 — Draw a Line. Which postulate lets us draw a straight line between two points?

Step 2: Prop 1 — To construct an equilateral triangle on a given finite straight line.. Which earlier proposition constructs an equilateral triangle?

Step 3: P2 — Extend a Line. Which postulate lets us extend a line segment?

Step 4: P3 — Draw a Circle. Which postulate lets us draw a circle?

Step 5: D15 — Circle. What definition tells us all radii of a circle are equal?

Step 6: P3 — Draw a Circle. Same tool as step 4 — another circle.

Step 7: D15 — Circle. What definition tells us all radii of a circle are equal?

Step 8: CN3 — Subtraction Preserves Equality. If equals are subtracted from equals, the remainders are equal.

Step 9: CN1 — Transitivity of Equality. Things equal to the same thing are equal to each other.

Part C: Thinking Deeper — Model Answers

Question 1: Look at Postulate 3: what operation does it actually permit? Why doesn't it directly allow copying a length?

> **Teacher note:** Postulate 3 permits drawing a circle with a given center passing through a given point. Crucially, it does not permit setting a compass to a fixed width and moving it elsewhere. Euclid's compass "collapses" when lifted from the paper. This is why copying a length requires an elaborate construction rather than a simple compass transfer.

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Question 2: How does the equilateral triangle help us transfer the length?

> **Teacher note:** The equilateral triangle DAB provides two equal arms: DA = DB. Since D is equidistant from both A and B, it serves as a relay point. A circle centered at D can reach the same distance in both directions — toward A and toward B. By adjusting the radius of this circle (using the length from B's circle), we effectively redirect the length BC toward point A.

Question 3: What does this proposition tell us about the power of Euclid's postulates?

> **Teacher note:** It tells us that Euclid's five postulates, though minimal, are sufficient to perform operations that seem to require more powerful tools. Even though P3 only gives us a "collapsing compass," Prop 2 proves we can still copy lengths anywhere. This is a hallmark of a well-chosen axiom set: it appears limited but can derive everything needed.

Part D: Challenge Problem — Model Answer

> **Teacher note:** If point A lies on segment BC, or even coincides with B, the construction still works in principle — the equilateral triangle, extensions, and two circles can still be drawn. However, some degenerate cases require care: if A = B, then the equilateral triangle collapses and the construction is trivial (the line is already at the point). This edge-case thinking is valuable because it tests whether students understand the construction as a general method rather than a single picture.

Notes: Common Student Mistakes

• Confusing P1 (draw a line between two points) with P2 (extend an existing line). Step 1 uses P1 to connect A and B. Step 3 uses P2 to extend lines DA and DB. Students often swap these.

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• Using CN1 where CN3 is needed. The key step involves subtracting equal lengths (DA from DL, and DB from DG) to get equal remainders (AL and BG). This is CN3 (equals subtracted from equals yield equal remainders), not CN1 (transitivity).

• Citing Prop 1 for step 1 instead of P1. Step 1 merely joins points A and B with a line segment — that is Postulate 1. Proposition 1 is invoked in step 2 to construct the equilateral triangle on segment AB.

• Failing to see why two circles are needed. Some students want to draw a single circle to transfer the length. The two-circle approach is necessary because our compass collapses — we cannot "set" it to length BC and redraw at A.