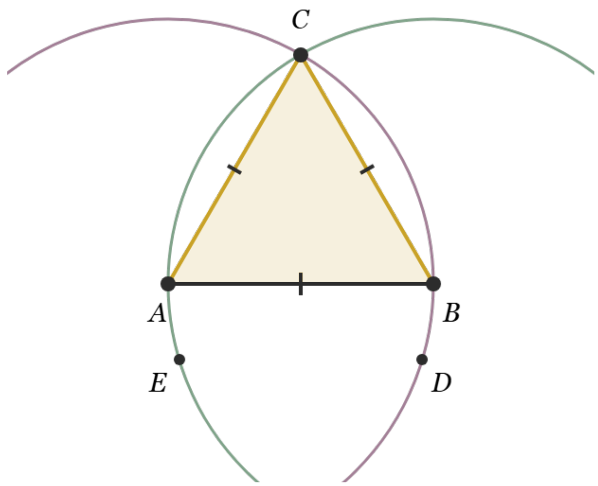
Lesson Plan: Proposition 1

To construct an equilateral triangle on a given finite straight line.

Grade: 9-12	Duration: 45-55 minutes	Type: Construction



In this lesson, you'll construct an equilateral triangle on a given line segment using only a compass and straightedge. This is the very first proposition in Euclid's Elements — and there's a reason it comes first. Two overlapping circles naturally create a point that's the same distance from both centers, and that's all you need. No measuring, no guessing. The equality comes straight from the definition of a circle.

Stage 1: Desired Results (UbD)

Essential Question

How can we guarantee that a geometric figure has exact properties using only the simplest tools?

Learning Objectives

1. Construct an equilateral triangle on a given segment using compass and straightedge
2. Explain why the intersection of two circles with equal radii produces an equidistant point
3. Identify and apply the definitions of circle (D15) and equilateral triangle (D20)
4. Write a step-by-step construction proof using Euclid's postulates and common notions

Key Understandings

• A compass and straightedge are sufficient to construct perfectly equal lengths without measurement

• The intersection point of two circles centered at opposite ends of a segment is equidistant from both endpoints

• Euclid's Common Notion 1 (things equal to the same thing are equal to each other) bridges the gap between two separate radius equalities

Vocabulary

Equilateral triangle, Radius, Circle, Postulate, Common notion, Construction, Straightedge, Compass

---

Stage 2: Assessment Evidence

Formative Checks

• Watch for students who draw arcs instead of full circles — have them draw the complete circle so they can see both intersection points

• Ask pairs to explain in their own words why the triangle must be equilateral

• Exit slip: given a new segment, list the construction steps from memory

Materials Needed

• Compass and straightedge for each student

• Blank unlined paper

• Printed worksheet with pre-drawn segments of varying lengths

• Colored pencils (optional, for marking equal lengths)

Stage 3: Learning Plan

Warm-Up (5 minutes)

Draw a segment AB on the board (or have your student draw one). Pose the challenge: 'Using only an unmarked ruler and a compass, how would you draw a triangle where all three sides are exactly the same length? No measuring allowed.' Give a couple of minutes to think about it. Don't confirm or deny any ideas yet — just collect them.

Direct Instruction (12 minutes)

Walk through the construction step by step. (1) Start with segment AB. (2) Put the compass point on A, open it to B, and draw a full circle. (3) Now put the compass point on B, open it to A, and draw another full circle. (4) Label a point C where the two circles cross. Here's the key insight: AC = AB because they're both radii of the circle centered at A — every point on a circle is the same distance from the center (that's Definition 15). For the same reason, BC = BA. And since AC = AB and BC = AB, Common Notion 1 tells us AC = BC too. All three sides equal. That's it — no measurement needed.

Guided Practice (12 minutes)

Now it's your student's turn. Have them do the construction on their own paper with a different starting segment length. Once they've drawn the triangle, have them label every equal length and write a short justification: which definition tells us the radii are equal? Which common notion chains the equalities together? If working with a partner, have each person use a different segment length — does the method still work?

Independent Practice (13 minutes)

Work through the worksheet: (a) Construct equilateral triangles on three segments of different lengths. (b) The two circles intersect in two points — does it matter which one you pick for C? Why or why not? (c) Challenge: can you construct an equilateral triangle if you only have a compass and no segment to start with? Explain your reasoning.

Closure (8 minutes)

Have your student narrate their construction aloud — every step and why it works. Then look ahead together: 'We can now build an equilateral triangle anywhere. But what if we need to copy a specific length to a completely different location — can the compass alone do that?' That question is exactly what Proposition 2 answers.

Differentiation: Supporting All Learners

For Struggling Students

Start with a pre-printed diagram showing both circles already drawn and labeled. Have your student trace over it, highlight the equal radii with colored pencils, and fill in: 'AC = AB because ___ . BC = AB because ___ . So AC = BC by ___ .'

For Advanced Students

The circles intersect at two points. Prove the construction works for both. Then investigate: how many distinct equilateral triangles can you construct on a single segment? (Two — one on each side.) Can you extend this to build a rhombus?

Tips: Teacher Notes

> **Teacher note:** Insist on full circles, not just arcs, for the first construction. Seeing both intersection points builds intuition for why the construction works and prepares students for later propositions.

> **Teacher note:** If compass slipping is an issue, press firmly and rotate slowly. Safety compasses with locking mechanisms help too.

> **Teacher note:** This is a good moment to establish what 'proof' means in this course: explaining WHY something works, not just showing HOW to do it. The construction is the how; the justification with definitions and common notions is the why.

Preview: Next Lesson

Next: Proposition 2 — To place a straight line equal to a given straight line with one end at a given point.

Given a line segment and a point somewhere else, construct a new line segment starting at that point that's exactly the same length as the given one.