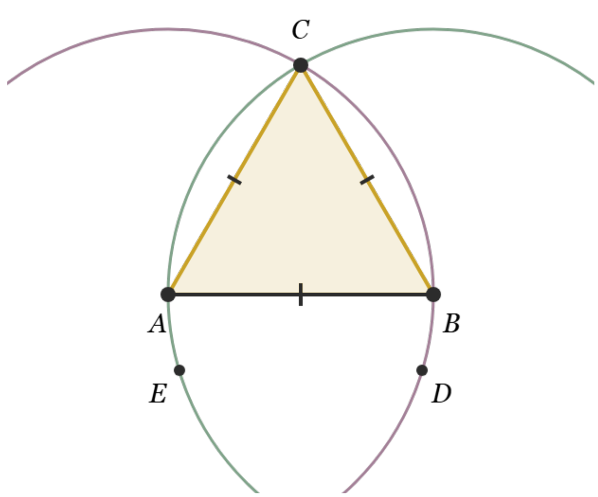
Answer Key: Proposition 1

To construct an equilateral triangle on a given finite straight line.

Part A: Construction Practice — Notes

Solution Figure



Teacher note: Students draw two circles: one centered at A passing through B (P3), one centered at B passing through A (P3). Point C is either intersection of the two circles. Guide students to label carefully — the proof depends on identifying AC and BC as radii of specific circles. Emphasize that we never measure anything; equal lengths come from the definition of a circle (D15). Have compasses and straightedges ready, and remind students that the straightedge has no markings.

Part B: Proof Justification — Answers

Each step is matched with the correct toolkit justification.

Available Toolkit:

- P1 — Draw a Line
- P3 — Draw a Circle
- D15 — Circle
- D20 — Types of Triangles (by sides)
- CN1 — Transitivity of Equality
- P2 — Extend a Line
- CN2 — Addition Preserves Equality

Step	Statement	Justification
1	Draw a circle centered at A passing through B	P3
2	Draw a circle centered at B passing through A	P3
3	Mark point C where the circles intersect. Draw lines AC and BC.	P1
4	AC = AB (both are radii of the circle centered at A)	D15
5	BC = BA (both are radii of the circle centered at B)	D15
6	AC = AB and BC = AB, therefore AC = BC	CN1
7	Since AC = AB = BC, triangle ABC is equilateral	D20

Teacher note: The proof works because every point on a circle is equidistant from the center (D15). AC equals AB because both are radii of the circle centered at A. BC equals BA because both are radii of the circle centered at B. Since AC = AB and BC = AB, Common Notion 1 (things equal to the same thing are equal to each other) gives us AC = BC. All three sides are equal, satisfying D20 (definition of equilateral triangle). The construction uses only P1 (join two points), P3 (draw a circle), D15 (circle definition), D20 (equilateral definition), and CN1 (transitivity of equality) — seven steps total.

Why each justification works:

Step 1: P3 — Draw a Circle. Which postulate lets us draw a circle?

Step 2: P3 — Draw a Circle. Same tool as step 1 — we're making another circle.

Step 3: P1 — Draw a Line. Which postulate lets us draw a straight line between two points?

Step 4: D15 — Circle. What definition tells us about radii of a circle?

Step 5: D15 — Circle. Same reasoning as step 4 — for the other circle.

Step 6: CN1 — Transitivity of Equality. Things equal to the same thing are equal to each other.

Step 7: D20 — Types of Triangles (by sides). What definition tells us what 'equilateral' means?

Part C: Thinking Deeper — Model Answers

Question 1: Why do you think Euclid chose this as Proposition 1? What makes it a good starting point?

> **Teacher note:** Euclid chose this as Proposition 1 because it is the simplest construction that produces a useful result from the postulates alone. It requires only P1, P3, and basic definitions — no previously proven propositions. It also establishes the equilateral triangle as a foundational tool that later propositions (especially Prop 2) depend on.

Question 2: Could you construct an equilateral triangle with just a straightedge (no compass)? Why or why not?

> **Teacher note:** No. A straightedge alone gives us P1 (join two points) and P2 (extend a line), but no way to guarantee equal distances. The compass and P3 are essential because a circle is the only tool that creates points at a known distance from a center. Without circles, we cannot ensure any two segments are equal.

Question 3: The two circles intersect at two points. Why does it not matter which one we pick for C?

> **Teacher note:** Both intersection points lie on both circles, so each one is simultaneously a radius of circle A (making it equidistant from A as B is) and a radius of circle B (making it equidistant from B as A is). The proof applies identically to either point. Choosing the other intersection simply produces a mirror-image equilateral triangle on the opposite side of line AB.

Part D: Challenge Problem — Model Answer

> **Teacher note:** Applying Proposition 1 again using side AC as the base produces a second equilateral triangle ACD sharing side AC with the original triangle ABC. The result is a rhombus (diamond shape) with vertices A, B, C, D, where all four outer sides are equal. This demonstrates that Prop 1 can be applied repeatedly to any segment — including segments produced by earlier constructions — to build more complex figures from simple equilateral building blocks.

Notes: Common Student Mistakes

• Confusing P1 (draw a straight line between two points) with P3 (draw a circle). Students often cite P1 when drawing circles. Remind them: P1 is for lines, P3 is for circles.

• Writing D15 when D20 is needed for the final step. D15 tells us radii are equal; D20 tells us what "equilateral" means. The last step invokes D20 to conclude the triangle is equilateral.

• Assuming the two circles must intersect without justification. Euclid himself does not explicitly prove this — it follows from an unstated continuity assumption. If advanced students raise this, acknowledge it as a genuine gap that later mathematicians addressed.

• Labeling point C but not explaining why it exists. Students should state that C is the intersection of the two circles, not just a point they chose arbitrarily.